DAY TWENTY TWO

Magnetism

Learning & Revision for the Day

Current Loop as a Magnetic Dipole

A current loop is equivalent to a magnetic dipole. If $A(=\pi a^2)$ be the area of the loop, then the magnitude of its dipole moment is

$$p_m = iA = i\pi a^2$$

where, a is radius of coil, i is current flowing through it

$$i = \frac{p_m}{\pi a^2} \quad \text{...(i)}$$

Magnetic field at the centre of a circular current loop is given by

$$B = \frac{\mu_0 i}{2a} \quad \text{...(ii)}$$

Putting the value of i from Eq. (i) in Eq. (ii), we get

$$B = \frac{\mu_0}{2\pi}\frac{p_m}{a^3}$$

This is the expression for the magnetic field at the centre of the current loop in terms of its dipole moment. Instead of circular loop, if there is a circular coil having n turns, its dipole moment would be $p_m = niA = ni\pi a^2$.

Bar Magnet

A bar magnet may be viewed as a combination of two magnetic poles, North pole and South pole, separated by some distance. The distance is known as the magnetic length of the given bar magnet.

A bar magnet exhibits two important properties, namely

(i) the attractive property and (ii) the directive property.

- If m is the pole strength and $2\mathbf{l}$ the magnetic length of the bar magnet,then its magnetic moment is $\mathbf{M} = m(2\mathbf{l})$.
- Magnetic moment is a vector quantity whose direction is from S pole towards N pole.
- SI unit of magnetic pole strength (m), is **ampere metre** (Am) and of magnetic dipole moment $(\mathbf{M})$ is **ampere metre^2** (Am^2).
- If a bar magnet is broken, the fragments are independent magnetic dipoles and not isolated magnetic poles.

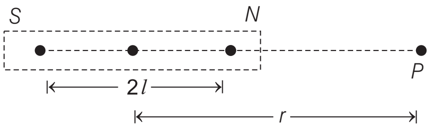
Magnetic Field due to a Bar Magnet

The magnetic field in free space, at a point having distance r from the given bar magnet (or magnetic dipole) is calculated in two conditions, along axial line and along equatorial line.

- **Along axial line** $\mathbf{B} = \frac{\mu_0}{4\pi} \frac{2\mathbf{M}r}{(r^2 - l^2)^2}$

  and the direction of **B** is the same as the direction of **M**. For a short dipole (or for a far away point on the axis) when $r >> l$, the above relation is simplified as

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{2\mathbf{M}}{r^3}$$



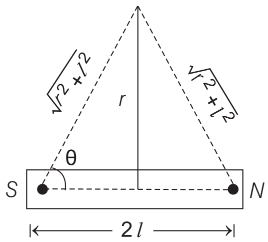
- **Along the equatorial line**

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{\mathbf{M}}{(r^2 + l^2)^{3/2}}$$

  If $r >> l$, the relation is modified as, $\mathbf{B} = \frac{\mu_0}{4\pi} \frac{\mathbf{M}}{r^3}$

  However, along the equatorial line,

  the direction of **B** is opposite to that of **M**.



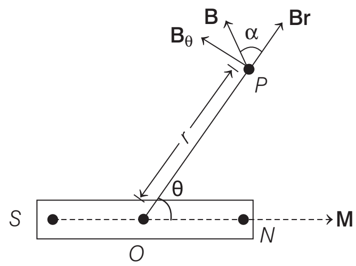
- In general, in a direction making an angle θ from with the magnetic axis, the magnetic field is given by

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{\mathbf{M}}{r^3} \sqrt{(3\cos^2\theta + 1)}$$

  In these relations, μ_0 is a constant having a value of $4\pi \times 10^{-7}$ T mA^{-1} and it is known as the **magnetic permeability of free space.**

  For solenoid $\mathbf{B} = \mu_0 ni$

  where, n is number of turns per unit length of solenoid and i the current through it.



Bar Magnet as an Equivalent Solenoid

The magnetic field (axial) at a point at a distance r and radius a of solenoid is given by $\mathbf{B} = \frac{\mu_0 nIla^2}{r^3}$

and magnetic moment of solenoid is $\mathbf{M} = n(2l)I\pi a^2$

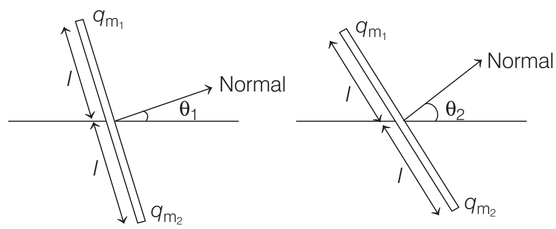
NOTE
- Even a single electron moving in its orbit behaves as a magnetic dipole and has a definite magnetic moment.
- Bohr magnetron is the magnetic moment due to the orbital motion of an electron revolving in the inner most orbit ($n = 1$). Its value is $m_B = \frac{eh}{4\pi m_e} = 9.27 \times 10^{-24}$ A-m^2

Torque on a Magnetic Dipole in a Magnetic Field

A magnetic dipole when placed in an uniform magnetic field, does not experience any net force. However, it experiences a torque given by $\tau = \mathbf{M} \times \mathbf{B}$ or $\tau = MB\sin\theta$

where, θ is the angle from the magnetic field, along which the dipole has been placed.

- **Work done** in rotating a magnetic dipole in a uniform magnetic field **B** from an initial orientation θ_1 to the final orientation θ_2, is given by $W = MB(\cos\theta_1 - \cos\theta_2)$.



Initial position fig. (i) Final position fig. (ii)

- **Potential energy** of a magnetic dipole placed in a uniform magnetic field, is given by $U_B = -\mathbf{M} \cdot \mathbf{B} = -MB\cos\theta$ where, θ is the angle from the direction of magnetic field and the axis of dipole.
- **The magnetic compass** (needle) of magnetic moment M and moment of inertia I and allowing it to oscillate in the magnetic field. Then, its time-period is $T = 2\pi\sqrt{I/MB}$
- Behaviour of a magnetic dipole in a magnetic field, is similar to the behaviour of an electric dipole in an electric field. However, the constant $\frac{1}{4\pi\varepsilon_0}$ is replaced by $\frac{\mu_0}{4\pi}$.
- If a magnetic dipole is in the form of a wire or a thin rod, when bent, its magnetic dipole moment **M** changes because the separation between its poles has changed.

Magnetic Field Lines

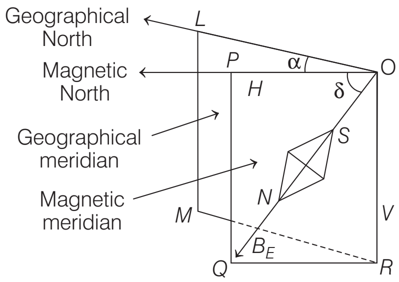
The magnetic field lines is defined as the path along which the compass needles are aligned. They are used to represent magnetic field in a region.

- Magnetic field lines are closed continuous curves.
- Tangent drawn at any point on magnetic field lines gives the direction of magnetic field at that point.
- Two magnetic field lines cannot intersect each other.
- Outside a magnet, they are directed from north to south pole and inside a magnet they are directed from south to north.

The Earth's Magnetism

The earth is a natural source of magnetic field, thus a magnetic field is always present everywhere near the surface of the earth. A freely suspended magnet always points in the north-south direction even in the absence of any other magnet. This suggests that the earth itself behaves as a magnet which causes a freely suspended magnet (or magnetic needle) to point always in a particular direction : north and south. The shape of earth's magnetic field resembles that of a bar magnet of length one-fifth of earth's diameter buried at its centre.

Magnetic field of earth is shown in the figure given below.



Magnetic Elements of Earth

- **Angle of Declination (α)** At a given place, the acute angle between the magnetic meridian and the geographical meridian is called the angle of declination (or magnetic declination) α at that place.
- **Angle of Inclination or Dip (δ)** The angle of dip δ at a place is the angle which the direction of the earth's total magnetic field B_E subtends with the horizontal direction.
- **Horizontal Component of the Earth's Magnetic Field (B_H)** As earth's magnetic field, in general, is inclined at an angle δ with the horizontal direction, it may be resolved into horizontal component B_H and a vertical component B_V, where $B_H = B_E \cos\delta$ and $B_V = B_E \sin\delta$

$$\Rightarrow \quad B_E = \sqrt{B_H^2 + B_V^2} \quad \text{and} \quad \tan\delta = \frac{B_V}{B_H}$$

Variation of Magnetic Elements of the Earth

At the magnetic equator, angle of dip is zero. Value of the angle of dip gradually increases, on going from equator to magnetic poles. At the magnetic poles, value of the dip angle is 90°.

At the magnetic equator, $B_H = B_E \cos 0° = B_E$

and at poles, $B_H = B_E \cos 90° = 0$.

Similarly, at the magnetic equator, $B_V = B_E \sin 0° = 0$

and at the poles, $B_V = B_E \sin 90° = B_E$.

Magnetic elements of the earth at a place change with time also.

Neutral Points

A neutral point is a point at which the resultant magnetic field is zero. Following two cases are of special interest.

1. When a bar magnet is placed along the magnetic meridian with its North pole pointing towards geographical North, two neutral points are obtained on either side of the magnet along its equatorial line. If r be the distance of the neutral point, then $\frac{\mu_0}{4\pi}\frac{M}{r^3} = B_H$.
2. When a bar magnet is placed along the magnetic meridian, with its North pole pointing towards the geographical South, two neutral points are obtained on either side of the magnet along its axial line.

   Hence, we have $\frac{\mu_0}{4\pi}\frac{2M}{r^3} = B_H$.

Tangent Galvanometer

It is an instrument to measure electric current. The essential parts are a vertical coil of conducting wire and a small compass needle pivoted at centre of coil. The deflection, θ of needle is given by,

$$\tan\theta = \frac{B}{B_H} \Rightarrow B_H \tan\theta = \frac{\mu_0 IN}{2r}$$

or
$$i = \frac{2r B_H}{\mu_0 N}\tan\theta = K\tan\theta$$

Magnetisation of Materials

There are some substances/materials which acquire magnetic properties on placing them in magnetic field. This phenomena is called magnetisation of materials.

To describe the magnetic properties of material, we have to understand the following terms:

(i) **Magnetic Induction or Magnetic Flux Density (B)** Whenever a piece of magnetic substance is placed in an external magnetising field, the substance becomes magnetised. If $\mathbf{B}_0$ is the magnetic field in free space, then

$$\mathbf{B} = \mu_r \mathbf{B}_0.$$

$\oint \mathbf{B}\cdot d\mathbf{S}$ is magnetic flux which is equal to $\mu_0 m_{\text{inside}}$, where m_{inside} is the net pole strength inside a close surface.

(ii) **Magnetic Permeability (μ)** It is the degree or extent to which the magnetic lines of induction may pass through a given distance.

Magnetic permeability of free space μ_0 has a value of $4\pi \times 10^{-7}$ TmA^{-1}. However, for a magnetic material, absolute permeability (μ) has a value, different than μ_0.

For any magnetic substance, $\frac{\mu}{\mu_0} = \frac{\mathbf{B}}{\mathbf{B}_0} = \mu_r$ = relative magnetic permeability of that substance. **Relative magnetic permeability** μ_r is a unitless and dimensionless term.

(iii) **Intensity of Magnetisation (*I*)** Intensity of magnetisation of a substance is defined as the magnetic moment induced in the substance per unit volume, when placed in the magnetising field. Thus, $\mathbf{I} = \frac{\mathbf{M}}{V}$

It is a vector quantity and its SI unit is Am^{-1}.

(iv) **Intensity of Magnetising Field or Magnetic Intensity (*H*)** It is a measure of the capability of external magnetising field to magnetise the given substance and is mathematically defined as

$$\mathbf{H} = \frac{\mathbf{B}_0}{\mu_0} \text{ or } \mathbf{H} = \frac{\mathbf{B}}{\mu}$$

Magnetic intensity **H** is a vector quantity and its SI unit is Am^{-1}.

(v) **Magnetic Susceptibility (χ_m)** Magnetic susceptibility of a substance is the ratio of the intensity of magnetisation *I* induced in the substance to the magnetic intensity *H*. Thus, $\chi_m = \frac{I}{H}$. It is a scalar quantity and it has no units or dimensions.

Relation between μ_r and χ_m we have, $B = \mu_0(I + H)$

or $B = \mu_0 H \left(\frac{I}{H} + 1\right)$ or $B = B_0(\chi_m + 1)$ or $\frac{B}{B_0} = \chi_m + 1$

But $\frac{B}{B_0} = \frac{\mu}{\mu_0} = \mu_r$ = relative permeability

$\therefore \quad \mu_r = \chi_m + 1$

Magnetic Materials

According to behaviour of magnetic substances, they are classified into three cases:

Diamagnetic Materials

These are materials which show a very small decrease in magnetic flux, when placed in a strong magnetising field. Hydrogen, water, copper, zinc, antimony, bismuth, etc., are examples of diamagnetic materials.

- In a diamagnetic material, the net magnetic moment (sum of that due to orbital motion and spin motion of electrons) of an atom is zero. The external magnetic field **B** distorts the electron orbit and thus induces a small magnetic moment in the opposite direction.
- Diamagnetic materials are feebly repelled in an external magnetic field and thus have a tendency to shift from the stronger to weaker regions of the magnetic field.
- The relative permeability of any diamagnetic substance is slightly less than 1 (i.e. $\mu_r < 1$) and susceptibility has a small negative value.
- Diamagnetism is an intrinsic property and does not vary with magnetic field **B** or temperature.

Paramagnetic Materials

These are the materials which show a small increase in the magnetic flux when placed in a magnetising field. Oxygen, air, platinum, aluminium, etc., are examples of paramagnetic materials.

- In a paramagnetic material, the net magnetic moment of every atom is non-zero.
- Paramagnetic materials are feebly attracted in an external magnetic field and thus, have a tendency to shift from the weaker to the stronger regions of magnetic field.
- The relative permeability μ_r of a paramagnetic material is slightly greater than one ($\mu_r > 1$). Magnetic susceptibility χ_m of paramagnetic materials is positive.
- Paramagnetism is temperature dependent. According to the **Curie's law,** the magnetic susceptibility of a paramagnetic substance is inversely proportional to its temperature *T*.

  Mathematically, $\chi_m = \frac{C}{T}$, where *C* is the **Curie constant**.

Ferromagnetic Materials

These are the materials which are strongly attracted by a magnetic field and can themselves be magnetised even in a weak magnetising field. Iron, steel,nickel and cobalt are ferromagnetic.

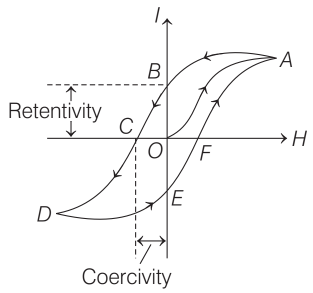
- These materials show a large increase in the magnetic flux, when placed in a magnetic field. Thus, for them $\mu_r >> 1$. Accordingly, χ_m is positive and large.
- Ferromagnetic materials exhibit all properties exhibited by paramagnetic substances and by a much larger measure.
- Magnetic susceptibility of ferromagnetic materials decreases steadily with a rise in temperature. Above a certain temperature T_c (known as **Curie temperature**), the substance loses its ferromagnetic character and begins to behave as a paramagnetic substance.
- Above the Curie temperature T_c, the magnetic susceptibility of a ferromagnetic material varies as

  $$\chi_m \propto \frac{1}{(T - T_c)} \quad \text{or} \quad \chi_m = \frac{C}{(T - T_c)}$$

  where, *C* is a constant. It is known as the Curie-Weiss law.

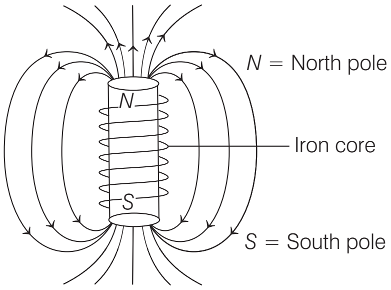
Hysteresis Curve

The lag of intensity of magnetisation behind the magnetising field during the process of magnetisation and demagnetisation of a ferromagnetic material is called **hysteresis**. The whole graph *ABCDEFA* is a closed loop and known as **hysteresis loop.**



Electromagnet

Electromagnets are usually in the form of iron core solenoids. The ferromagnetic property of the iron core causes the internal magnetic domains of the iron to line up with the smaller driving magnetic field produced by the current in the solenoid.



The effect is the multiplication of the magnetic field by factors of ten to eleven thousands. The solenoid field relationship is $B = k\mu_0 nI$, where $\mu = k\mu_0$ and k is the relative permeability of the iron, the figure shows the magnetic effect of the iron core.

Permanent Magnet

Substances which at room temperature retain their ferromagnetic property for a long period of time are called permanent magnets. Permanent magnets can be made in a variety of ways.

An efficient way to make a permanent magnet is to place a ferromagnetic rod in a solenoid and pass a current. The magnetic field of the solenoid magnetises the rod.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

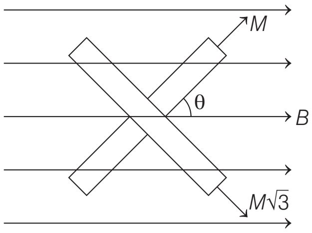
1 A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0.16 T, experiences a torque of magnitude 0.032 J. The magnetic moment of the bar magnet will be

(a) 0.23 JT^{-1} (b) 0.40 JT^{-1}
(c) 0.80 JT^{-1} (d) zero

2 A bar magnet of length 10 cm and having pole strength equal to 10^{-3} Wb, is kept in a magnetic field having magnetic induction B equal to $4\pi \times 10^{-3}$ T. It makes an angle of 30° with the direction of magnetic induction. The value of the torque acting on the magnet is

(a) 0.5 N-m (b) $2\pi \times 10^{-5}$ N-m
(c) $\pi \times 10^{-5}$ N-m (d) 0.5×10^{2} N-m

3 M and $M\sqrt{3}$ are the magnetic dipole moments of the two magnets, which are joined to form a cross figure. The inclination of the system with the field, if their combination is suspended freely in a uniform external magnetic field B is



(a) $\theta = 30°$ (b) $\theta = 45°$ (c) $\theta = 60°$ (d) $\theta = 15°$

4 A coil of 50 turns and area 1.25×10^{-3} m^2 is pivoted about a vertical diameter in an uniform horizontal magnetic field and carries a current of 2 A. When the coil is held with its plane in the N-S direction, it experiences a couple of 0.04 Nm, and when its plane is along the East-West direction, it experiences a couple of 0.03 Nm. The magnetic induction is

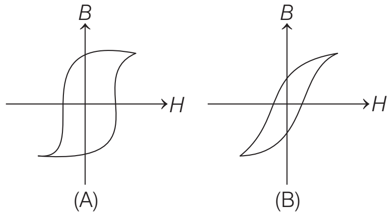
(a) 0.2 T (b) 0.3 T (c) 0.5 T (d) 0.4 T

5 A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is

→ JEE Main 2017 (Offline)

(a) 8.89 s (b) 6.98 s
(c) 8.76 s (d) 6.65 s

6 Hysteresis loops for two magnetic materials A and B are as given below



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then, it is proper to use

→ JEE Main 2016 (Offline)

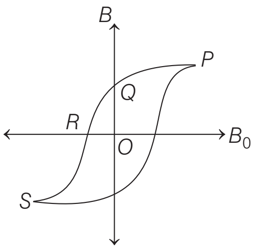
(a) A for electric generators and transformers
(b) A for electromagnets and B for electric generators
(c) A for transformers and B for electric generators
(d) B for electromagnets and transformers

7 The coercivity of a small magnet where the ferromagnet gets demagnetised is 3×10^{3} Am^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetised when inside the solenoid is

→ JEE Main 2014

(a) 30 mA (b) 60 mA (c) 3 A (d) 6 A

8 The figure illustrates how B, the flux density inside a sample of unmagnetised ferromagnetic material, varies with B_0, the magnetic flux density in which the sample is kept. For the sample to be suitable for making a permanent magnet.



(a) OQ should be large, OR should be small
(b) OQ and OR should both be large
(c) OQ should be small and OR should be large
(d) OQ and OR should both be small

9 Match the following columns.

	Column I		Column II
A.	Magnetic moment	1.	$[ML^0T^{-2}A^{-1}]$
B.	Permeability	2.	Vector
C.	Intensity of magnetisation	3.	Nm^3 / Wb
D.	Magnetic induction	4.	Scalar

Codes

	A	B	C	D		A	B	C	D
(a)	1	2	3	4	(b)	3	4	2	1
(c)	4	3	1	2	(d)	2	1	3	4

10 A current carrying coil is placed with its axis perpendicular to N-S direction. Let horizontal component of earth's magnetic field be H_0 and magnetic field inside the loop be H. If a magnet is suspended inside the loop, it makes angle θ with H. Then, θ equal to

(a) $\tan^{-1}\left(\frac{H_0}{H}\right)$ (b) $\tan^{-1}\left(\frac{H}{H_0}\right)$
(c) $\text{cosec}^{-1}\left(\frac{H}{H_0}\right)$ (d) $\cot^{-1}\left(\frac{H_0}{H}\right)$

11 Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2, respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to (Horizontal component of the earth's magnetic induction is 3.6×10^{-5} Wb/m^2) **→ JEE Main 2013**

(a) 3.6×10^{-5} Wb / m^2 (b) 2.56×10^{-4} Wb / m^2
(c) 3.50×10^{-4} Wb / m^2 (d) 5.80×10^{-4} Wb / m^2

12 The magnetic susceptibility of a paramagnetic substance at –73°C is 0.0060, then its value at –173°C will be

(a) 0.0030 (b) 0.0120 (c) 0.0180 (d) 0.0045

13 Consider the two idealised systems (i) a parallel plate capacitor with large plates and small separation and (ii) a long solenoid of length $L >> R$, radius of cross-section. In (i) **E** is ideally treated as a constant between plates and zero outside. In (ii) magnetic field is constant inside the solenoid and zero outside. These idealised assumptions, however contradict fundamental laws as below

(a) case (i) contradicts Gauss' law for electrostatic fields
(b) case (ii) contradicts Gauss' law for magnetic fields
(c) case (i) agrees with $\oint \mathbf{E} \cdot d\mathbf{l} = 0$
(d) case (ii) contradicts $\oint \mathbf{H} \cdot d\mathbf{l} = I_{en}$

14 A paramagnetic sample shows a net magnetisation of 8Am^{-1} when placed in an external magnetic field of 0.6 T at a temperature of 4 K. When the same sample is placed in an external magnetic field of 0.2 T at a temperature of 16 K, the magnetisation will be

(a) $\frac{32}{3}$ Am^{-1} (b) $\frac{2}{3}$ Am^{-1} (c) 6Am^{-1} (d) 2.4 Am^{-1}

15 If the areas under the I-H hysteresis loop and B-H hysteresis loop are denoted by A_1 and A_2, then

(a) $A_2 = \mu_0 A_1$ (b) $A_2 = A_1$
(c) $A_2 = \frac{A_1}{\mu_0}$ (d) $A_2 = \mu_0^2 A_1$

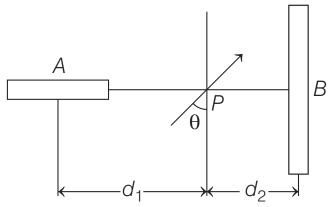
16 A short magnet oscillates with a time period 0.1 s at a place, where horizontal magnetic field is 24 μT. A downward current of 18 A is established in a vertical wire 20 cm East of the magnet. The new time period of oscillator

(a) 0.1 s (b) 0.089 s (c) 0.076 s (d) 0.057 s

17 Two bar magnets of the same mass, same length and breadth but having magnetic moments M and $2M$, respectively are joined together pole to pole and suspended by a string. The time period of the assembly in a magnetic field of strength H is 3 s. If now the polarity of one of the magnets is reversed and the combination is again made to oscillate in the same field, the time of oscillation is

(a) 3 s (b) $3\sqrt{3}$ s (c) $3/\sqrt{3}$ s (d) 6 s

18 Two magnets A and B are identical and these are arranged as shown in the figure. Their lengths are negligible in comparision to the separation between them. A magnetic needle is placed between the magnets at point P and it gets deflected by an angle θ. The ratio of distances d_1 and d_2, will be



(a) $(2 \cot \theta)^{1/3}$ (b) $(2 \tan \theta)^{1/3}$
(c) $(2 \cot \theta)^{-1/3}$ (d) $(2 \tan \theta)^{-1/3}$

19 The plane of a dip circle is set in the geographic meridian and the apparent dip is δ_1. It is then set in a vertical plane perpendicular to the geographic meridian.

The apparent dip angle is δ_2. The declination θ at the place is

(a) $\theta = \tan^{-1}(\tan\delta_1 \tan\delta_2)$ (b) $\theta = \tan^{-1}(\tan\delta_1 + \tan\delta_2)$

(c) $\theta = \tan^{-1}\left(\frac{\tan\delta_1}{\tan\delta_2}\right)$ (d) $\theta = \tan^{-1}(\tan\delta_1 - \tan\delta_2)$

20 This question contains two statements : Statement I (*Assertion*) *and Statement* II (*Reason*). *The question also has four alternative choices, only one of which is the correct answer. You have to select one of the codes* (*a*), (*b*), (*c*), (*d*).

Statement I A current carrying loop is free to rotate. It is placed in a uniform magnetic field. It attains equilibrium when its plane is perpendicular to the magnetic field.

Statement II The torque on the coil is zero when its plane is perpendicular to the magnetic field.

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I

(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I

(c) Statement I is true; Statement II is false

(d) Statement I is false; Statement II is true

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is **→ JEE Main 2018**

(a) 2 (b) $\sqrt{3}$

(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

2 The current on the winding of a toroid is 2A. It has 400 turns and mean circumferential length is 40 cm. With the help of search coil and charge measuring instrument the magnetic field is found to be 1T. The susceptibility is

(a) 100 (b) 290
(c) 398 (d) 397

3 The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2 s. The magnet is cut along its length into three equal parts and three parts are then placed on each other with their like poles together. The time period of this combination will be

(a) 2 s (b) $\frac{2}{3}$ s

(c) $2\sqrt{3}$ s (d) $\frac{2}{\sqrt{3}}$ s

4 A thin circular disc of radius R is uniformly charged with density $\sigma > 0$ per unit area. The disc rotates about its axis with a uniform angular speed ω. The magnetic moment of the disc is **→ AIEEE 2011**

(a) $2\pi R^4 \sigma\omega$ (b) $\pi R^4 \sigma\omega$

(c) $\frac{\pi R^4}{2}\sigma\omega$ (d) $\frac{\pi R^4}{4}\sigma\omega$

5 Consider the plane S formed by the dipole axis and the axis of the earth. Let P be point on the magnetic equator and in S. Let Q be the point of intersection of the geographical and magnetic equators. The declination and dip angles at P and Q are

(a) 0° and 11.3° (b) 0° and 0°
(c) 11.3° and 6.5° (d) 11.3° and 11.3°

6 A bar magnet has pole strength 3.6 A-m and length 12 cm. Its area of cross-section is 0.9 cm^2. The magnetic field B at the centre of the bar magnet is

(a) 6×10^{-3} T (b) 5×10^{-2} T
(c) 2.5×10^{-2} T (d) 2.5×10^{-8} T

7 A bar magnet suspended by a suspension fibre, is placed in the magnetic meridian with no twist in the suspension fibre. On turning the upper end of the suspension fibre by an angle of 120° from the meridian, the magnet is deflected by an angle of 30° from the meridian. Then, the angle by which the upper end of the suspension fibre has to be twisted, so as to deflect the magnet through 90° from the meridian is

(a) 270° (b) 240°
(c) 330° (d) 180°

8 A bar magnet 8 cm long, is placed in the magnetic meridian with the N pole, pointing towards the geographical North. Two neutral points, separated by a distance of 6 cm are obtained on the equatorial axis of the magnet. If $B_H = 3.2 \times 10^{-5}$ T, then the pole strength of the magnet is

(a) 5 A-cm^2 (b) 10 A-cm^2
(c) 2.5 A-cm^2 (d) 20 A-cm^2

9 There are two current carrying planer coils made each from wire of length L. C_1 is circular coil (radius R) and C_2 is square (side a). These are so constructed that they have same frequency of oscillation when they are placed in the same uniform magnetic field **B** and carry the same current. The value of a in terms of R is

(a) $3R$ (b) $\sqrt{3}R$
(c) $\sqrt{2}R$ (d) $2R$

ANSWERS

SESSION 1	**1** (b)	**2** (a)	**3** (c)	**4** (d)	**5** (d)	**6** (d)	**7** (c)	**8** (b)	**9** (b)	**10** (a)
	11 (b)	**12** (b)	**13** (b)	**14** (b)	**15** (a)	**16** (c)	**17** (b)	**18** (a)	**19** (c)	**20** (c)
SESSION 2	**1** (c)	**2** (d)	**3** (b)	**4** (d)	**5** (a)	**6** (b)	**7** (a)	**8** (a)	**9** (a)	

Hints and Explanations

SESSION 1

1 As, $M = \dfrac{\tau}{B \sin \theta}$

$= \dfrac{0.032}{0.16 \times \sin 30°} = 0.40 \text{ JT}^{-1}$

2 As, $\mu_0 m = 10^{-3}$ Wb

$m = \dfrac{10^{-3}}{\mu_0}$

Magnetic moment of the magnet,

$M = m \times 2l = \dfrac{10^{-3}}{\mu_0}(0.1) = \dfrac{10^{-4}}{\mu_0}$

Now, $\tau = MB \sin \theta$

$= \left(\dfrac{10^{-4}}{\mu_0}\right) \times 4\pi \times 10^{-3} \sin 30°$

$= 0.5$ N-m

3 Torque (τ) acting on the magnet (1) is

$\tau_1 = MB \sin\theta$

$\tau_2 = \sqrt{3}\, MB \sin\theta$

For equilibrium, $\tau_1 = \tau_2$

$\therefore \quad MB \sin\theta = \sqrt{3}\, MB \cos\theta$

$\Rightarrow \tan\theta = \sqrt{3} = \tan 60° \Rightarrow \theta = 60°$

4 Here, $N = 50$, $A = 1.25 \times 10^{-3}$ m^2,

$I = 2$A

$M = NIA$

$= 50 \times 2 \times 1.25 \times 10^{-3}$

$= 0.125$ A-m^2

If the normal to the face of the coil makes an angle θ with the magnetic induction B, the torque

$\tau = MB \cos\theta = 0.04 \qquad \ldots$(i)

Now, when the plane of the coil is turned through 90°, the torque becomes,

$\tau = MB \sin\theta = 0.03 \qquad \ldots$(ii)

Squaring and adding Eqs. (i) and (ii), we get

$\tau = 0.05 \Rightarrow MB = 0.05$

$\Rightarrow \quad B = \dfrac{0.05}{M} = \dfrac{0.05}{0.125} = 0.4$ T

5 Time period of oscillation is

$T = 2\pi\sqrt{\dfrac{I}{MB}}$

$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 0.01}} = 0.665$ s

Time taken to complete 10 oscillations

$= 10 \times 0.665 = 6.65$ s

Hence, time for 10 oscillations is $t = 6.65$ s.

6 Area of hysteresis loop is proportional to the net energy absorbed per unit volume by the material, as it is taken over a complete cycle of magnetisation. For electromagnets and transformers, energy loss should be low. i.e. thin hysteresis curves.

Also, $|B| \to 0$ when $H = 0$ and $|H|$ should be small when $B \to 0$.

7 For solenoid, the magnetic field needed to be magnetised the magnet.

$H = nI$

$\left[\text{where, } N = 100,\ l = 10\text{cm} = \dfrac{10}{100}\text{m} = 0.1\text{ m}\right]$

$H = \dfrac{N}{l} I \Rightarrow 3 \times 10^3 = \dfrac{100}{0.1} \times I$

$I = 3$ A

8 For making permanent magnet, the material should have high residual magnetism and high coercivity i.e. OQ and OR should be large.

9 A - SI unit of magnetic moment is Nm^3 / Wb.

B - Permeability (μ) is a scalar.

C - $I = \dfrac{M}{V} = \dfrac{\text{A}}{\text{m}^{-1}}$, it is a vector.

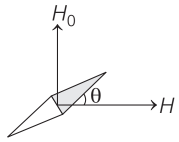
D - Magnetic induction

$B = \text{ML}^0\text{T}^{-2}\text{A}^{-1}$

10 In given case H and H_0 are perpendicular to each other.

From figure,

$\tan\theta = \dfrac{H_0}{H}$



$\Rightarrow \quad \theta = \tan^{-1}\left(\dfrac{H_0}{H}\right)$

11 $B_{\text{net}} = B_1 + B_2 + B_H$

$B_{\text{net}} = \dfrac{\mu_0}{4\pi}\dfrac{(M_1 + M_2)}{r^3} + B_H$

$= \dfrac{10^{-7}(1.2 + 1)}{(0.1)^3} + 3.6 \times 10^{-5}$

$= 2.56 \times 10^{-4}$ Wb/m^2

12 As, $\quad \chi_m \propto \dfrac{1}{T}$

$\Rightarrow \quad \dfrac{\chi_2}{\chi_1} = \dfrac{T_1}{T_2}$

or $\quad \dfrac{\chi_2}{0.0060} = \dfrac{273 - 73}{273 - 173} = \dfrac{200}{100} = 2$

or $\quad \chi_2 = 0.0120$

13 As Gauss' law states,

$\oint_S \mathbf{E} \cdot d\mathbf{S} = \dfrac{q}{\varepsilon_0}$ for electrostatic field. It does not contradict for electrostatic fields as the electric field lines do not form continuous closed path. According to Gauss's law in magnetic field,

$\oint_S \mathbf{E} \cdot d\mathbf{S} = 0$

It contradicts for magnetic field, because there is a magnetic field inside the solenoid and no field outside the solenoid carrying current but the magnetic field lines form the closed path.

14 As Curie's law explains, we can deduce a formula for the relation between magnetic field induction, temperature and magnetisation.

i.e. I (magnetisation) $\propto \dfrac{B \text{ (magnetic field induction)}}{t \text{(temperature in kelvin)}}$

$$\Rightarrow \quad \frac{I_2}{I_1} = \frac{B_2}{B_1} \times \frac{t_1}{t_2}$$

Let us suppose, here $I_1 = 8\,\text{Am}^{-1}$

$$B_1 = 0.6\text{T}, t_1 = 4\text{K}$$
$$B_2 = 0.2\text{T}, t_2 = 16\text{K}$$
$$\Rightarrow \quad \frac{0.2}{0.6} \times \frac{4}{16} = \frac{I_2}{8}$$
$$\Rightarrow \quad I_2 = \frac{2}{3}\,\text{Am}^{-1}$$

15 As, $B = \mu_0(H + I)$

$\Rightarrow \quad dB = \mu_0\, dH + \mu_0\, dI$

or $\oint HdB = \mu_0 \oint HdH + \mu_0 \oint H\, dI$

$\because \oint HdH = 0$

$\oint HdB = \mu_0 \oint H \cdot dI$

Area of the B-H loop

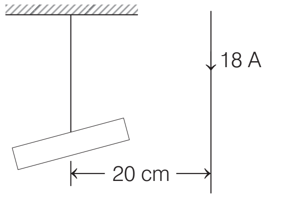
$= \mu_0 \times$ area of I-H loop

i.e. $\quad A_2 = \mu_0 A_1$

16 Given, $B_H = 24\mu\text{T}$

$$B_{\text{current}} = \frac{\mu_0 I}{2\pi d} = \frac{2 \times 10^{-7} \times 18}{0.2}$$
$$= 18\,\mu\text{ T}$$

Now, $T' = T\sqrt{\dfrac{24}{42}} = \dfrac{0.1 \times 2}{\sqrt{7}} = 0.076$ s



17 At the pole, for the combination

$$M_1 = 2M + M = 3M, T_1 = 3\text{ s}$$

When the polarity of one is reversed, then

$$M_2 = 2M - M = M$$

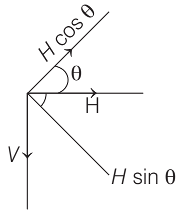
Thus, we have, from

$$\frac{T_2^2}{T_1^2} = \frac{M_1}{M_2} \quad \left(\because T = 2\pi\sqrt{\frac{I}{MB}}\right)$$
$$\Rightarrow \quad \frac{T_2^2}{T_1^2} = \frac{3M}{M} = 3$$
$$\therefore \quad T_2^2 = 3T_1^2 = 3 \times 9 = 27$$
$$\therefore \quad T_2 = \sqrt{27} = 3\sqrt{3}\text{ s}$$

18 $B = B_H \tan\theta$

$$\frac{\mu_0}{4\pi}\left(\frac{2M}{d_1^3}\right) = \left(\frac{\mu_0}{4\pi}\frac{M}{d_2^3}\right)\tan\theta$$
$$\frac{2}{d_1^3} = \frac{\tan\theta}{d_2^3}$$
$$\left(\frac{d_1}{d_2}\right)^3 = \frac{2}{\tan\theta} = 2\cot\theta$$
$$\frac{d_1}{d_2} = (2\cot\theta)^{1/3}$$

19 $\tan\delta_1 = \dfrac{V}{H\cos\theta}$



$$\tan\delta_2 = \frac{V}{H\cos(90° - \theta)} = \frac{V}{H\sin\theta}$$
$$\frac{\tan\delta_1}{\tan\delta_2} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$
$$\text{or} \quad \theta = \tan^{-1}\left(\frac{\tan\delta_1}{\tan\delta_2}\right)$$

20 The torque experienced by a coil in a magnetic field is,

$$\tau = M \times B = MB\sin\theta$$

when plane is perpendicular,

$$\theta = 90°$$
$$\tau = MB\sin 90° = MB \text{ [maximum]}$$

Hence, Statement I is true, while statement II is false.

SESSION 2

1 As $m = IA$, so to change dipole moment (current is kept constant), we have to change radius of loop.

Initially, $m = I\pi R^2$ and $B_1 = \dfrac{\mu_0 I}{2R_1}$

Finally, $m' = 2m = I\pi R_2^2$

$$\Rightarrow \quad 2I\pi R_1^2 = I\pi R_2^2$$
$$\text{or} \quad R_2 = \sqrt{2}R_1$$
$$\text{So,} \quad B_2 = \frac{\mu_0 I}{2(R_2)} = \frac{\mu_0 I}{2\sqrt{2}R_1}$$
$$\text{Hence, ratio } \frac{B_1}{B_2} = \frac{\left(\dfrac{\mu_0 I}{2R_1}\right)}{\left(\dfrac{\mu_0 I}{2\sqrt{2}R_1}\right)} = \sqrt{2}$$
$$\therefore \quad \text{Ratio } \frac{B_1}{B_2} = \sqrt{2}$$

2 As, $n = \dfrac{400}{2\pi R} = \dfrac{400}{40 \times 10^{-2}} = 1000$

$$\mu = ni = 1000 \times 2 = 2000$$
$$B = \mu_0 \mu_r \mu$$
$$\Rightarrow \quad \mu_0 \mu_r = \frac{1}{2000} = 5 \times 10^{-4}$$
$$\Rightarrow \quad \mu_r = \frac{5 \times 10^{-4}}{\mu_0} = \frac{5 \times 10^{-4}}{4\pi \times 10^{-7}} = 398$$
$$\Rightarrow \quad \chi = \mu_r - 1 = 397$$

3 The time period of oscillations of magnet,

$$T = 2\pi\sqrt{\left(\frac{I}{MH}\right)} \qquad \text{...(i)}$$

where, I = moment of inertia of magnet

$$= \frac{mL^2}{12}$$

[m, being the mass of magnet]

M = pole strength $\times L$

and H = horizontal component of the earth's magnetic field.

When the three equal parts of magnet are placed on one another with their like poles together, then

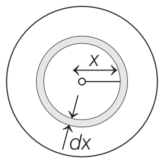
$$I' = \frac{1}{12}\left(\frac{m}{3}\right) \times \left(\frac{L}{3}\right)^2 \times 3$$
$$= \frac{1}{12}\frac{mL^2}{9} = \frac{I}{9}$$

and M' = pole strength $\times \dfrac{L}{3} \times 3 = M$

Hence, $T' = 2\pi\sqrt{\left(\dfrac{I/9}{MH}\right)}$

or $\quad T' = \dfrac{1}{3} \times T \quad$ or $T' = \dfrac{2}{3}$ s

4 Let us consider the disc to be made up of large number of concentric elementary rings. Consider one such ring of radius x and thickness dx. Charge on this elementary ring,



$$dq = \sigma \times 2\pi x dx = 2\pi\sigma x dx$$

Current associated with this elementary ring,

$$dI = \frac{dq}{dt} = dq \times f = \sigma\omega x dx$$

[$\because f$ is frequency and $\omega = 2\pi f$]

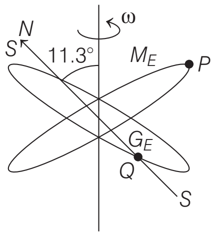
Magnetic moment of this elementary ring,

$$dM = d/\pi x^2 = \pi\sigma\omega x^3 dx$$

$\therefore$ Magnetic moment of the entire disc,

$$M = \int_0^R dM$$
$$= \pi\sigma\omega\int_0^R x^3 dx = \frac{1}{4}\pi R^4 \sigma\omega$$

5 P is in the plane S, needle is in North, so the declination is zero.



P is also on the magnetic equator, so the angle of dip = 0, because the value of angle of dip at equator is zero Q is also on the magnetic equator, thus the angle of dip is zero. As the earth tilted on its axis by 11.3°, thus the declination at Q is 11.3°.

6 As, $I = \dfrac{m}{A} = \dfrac{3.6}{0.9 \times 10^{-4}}$

$$= 4 \times 10^4 \text{ Am}^{-1}$$

$$H_N = \frac{m}{4\pi d^2}$$

$$= \frac{3.6}{4\pi \times (6 \times 10^{-2})^2}$$

$$= 79.6 \text{ Am}^{-1}$$

$$H = H_N + H_S$$

$\Rightarrow$ $H = H_N + H_S = 159.2$ A/m, towards S pole

$$B = \mu_0 (H + I) = 4\pi \times 10^{-7} (4 \times 10^4 - 159.2)$$

$= 5 \times 10^{-2}$ T, towards N pole

7 As, $\tau = C\phi = MH \sin\theta$

***Case* I**
$\theta = 30°, \phi° = 120° - 30° = 90°$ ...(i)

***Case* II** $C(\phi - 90°) = MH \sin 90°$ ...(ii)

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{\phi - 90°}{90°} = \frac{MH \sin 90°}{MH \sin 30°}$$

$$= \frac{1}{1/2} = 2$$

or $\phi - 90° = 180°$

or $\phi = 180° + 90° = 270°$

8 At the neutral point,

$$B = B_H$$

$$\Rightarrow \frac{\mu_0}{4\pi} \times \frac{M}{(r^2 + l^2)^{3/2}} = B_H$$

In CGS system,

$$\frac{M}{(r^2 + l^2)^{3/2}} = B_H$$

$$\Rightarrow \frac{m \times 2l}{(r^2 + l^2)^{3/2}} = B_H$$

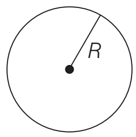
$$\Rightarrow \frac{m \times 2 \times 4}{(3^2 + 4^2)^{3/2}} = 0.32$$

$$\Rightarrow \frac{8m}{(25)^{3/2}} = 0.32$$

$$\Rightarrow m = \frac{125 \times 0.32}{8}$$

$$= 5 \text{ A-cm}^2$$

9



C_1 = Circular coil of radius R, length L, number of turns per unit length $n_1 = \dfrac{L}{2\pi R}$

Magnetic moment of C_1

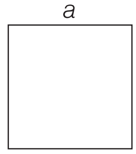
$$\Rightarrow m_1 = n_1 I A_1$$

$$m_1 = \frac{L \cdot I \cdot \pi R^2}{2\pi R},$$

$$m_1 = \frac{LIR}{2} \quad \text{...(i)}$$

Moment of inertia of C_1

$$\Rightarrow I_1 = \frac{MR^2}{2} \quad \text{...(ii)}$$



C_2 = Square of side a and perimeter L, number of turns per unit length $n_2 = \dfrac{L}{4a}$

Magnetic moment of C_2

$$\Rightarrow m_2 = n_2 I A_2$$

$$m_2 = \frac{L}{4a} \cdot I \cdot a^2$$

$$m_2 = \frac{LIa}{4} \quad \text{...(iii)}$$

Moment of inertia of C_2

$$\Rightarrow I_2 = \frac{Ma^2}{12} \quad \text{...(iv)}$$

Frequency of C_1

$$\Rightarrow f_1 = 2\pi \sqrt{\frac{I_1}{m_1 B}}$$

Frequency of C_2

$$\Rightarrow f_2 = 2\pi \sqrt{\frac{I_2}{m_2 B}}$$

According to question,

$$f_1 = f_2$$

$$2\pi \sqrt{\frac{I_1}{m_1 B}} = 2\pi \sqrt{\frac{I_2}{m_2 B}}$$

$$\frac{I_1}{m_1} = \frac{I_2}{m_2}$$

or $\dfrac{m_2}{m_1} = \dfrac{I_2}{I_1}$

Putting the values by Eqs. (i), (ii), (iii) and (iv), we get

$$\frac{LIa \cdot 2}{4 \times LIR} = \frac{Ma^2 \cdot 2}{12 \cdot MR^2}$$

$$3R = a$$

Thus, the value of a is $3R$.